SYLLABUS

Mathematics for Political Science

Political Science 598
Fall 2014
Mo/We 10:00-11:30am
4151 USB

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This course provides an introduction to a set of mathematical concepts and skills commonly used in formal and empirical methods in political science. These include: multivariate calculus techniques and applications; certain fundamental topics in the study of probability and distributions; common techniques in linear algebra; and a detailed examination of optimization. While a greater weight will be placed on preparing students for coursework on formal theory, most of the concepts examined are regularly employed in the study of empirical methods, and lectures and problem sets will use examples from both formal and empirical methods.

Three objectives will be pursued throughout the course. First, to develop fluency with all of the concepts and techniques described in detail in the biweekly meetings laid out below. Second, to increase comfort with the symbolic rendering of real world concepts, formal reasoning, and methods of proof. Third, to improve understanding of mathematics in both formal and quantitative studies in political science. In other words, to make following research which employs mathematics easier, and to provide you with a set of commonly used skills which will help in developing your own research.

Readings: There are two required books for the course:

Simon, Carl P., and Lawrence Blume. Mathematics for Economists. Vol. 7. New York: Norton, 1994.

Sundaram, Rangarajan K. A First Course in Optimization Theory. Cambridge: Cambridge University Press, 1996.

These texts will be abbreviated in the reading list using the abbreviations *SimBlu* and *Sun*, respecitively.

Course requirements: The course requirements comprise the following components.

Lecture and section participation: An important component of this course is active engagement with the material in both lecture and section. This also means benefitting from interacting with the instructors and fellow students. For this reason, attendance at all lectures and sections is mandatory, with the obvious exceptions of illness, personal issues or family emergency.

Problem sets: The course will have 8 problem sets, all of which must be submitted one week after they are released. The purpose of problem sets is to learn key concepts and their application through experience and repeated interaction. You should feel free to consult with your classmates (and the instuctors in office hours) but all final submitted answers should be fully understood and be your own work product. Problems sets will determine 50% of the course grade.

Midterm examination: There will be a midterm examination held in class on Wednesday, October 22nd. The midterm will cover a select set of the most important concepts discussed in the course up to that point. The emphasis will be on understanding and usage of the concepts, so you should focus your study on understanding the concepts and working through *brief* and memorable examples which illustrate how to employ particular techniques or ideas. The midterm will count towards 20% of the course grade.

Final examination: The final exam will be held on Monday, December 15th from 4-6pm. It will cover all of the material from the course, but with an emphasis on concepts covered after the midterm examination. The final exam will determine 30% of the course grade.

Office hours: My office hours will be held in 6741 Haven Hall on Mondays from 4:00-5:00pm and on Wednesdays from 2:30-4:00pm. If you cannot meet at this time, please send me an email and we will arrange a separate time to meet up.

Accommodations for Students with Disabilities: If you think you need an accommodation for a disability, please let me know as soon as possible. Some aspects of this course may be modified to facilitate your participation and progress. We can work with the Office of Services for Students with Disabilities to determine appropriate academic accommodations. SSD (tel.: 734-763-3000) typically recommends accommodations through a Verified Individualized Services and Accommodations (VISA) form. Any information you provide is private and confidential and will be treated as such.

Class topics and readings

I: Topics in Calculus and Probability

9/3: Differentiation

secant · tangent · definition of derivative · power rule · binomial theorem · binomial coefficients · derivative of e^x and $\ln x$ · product rule · chain rule · anti-derivatives · fundamental theorem of calculus

No assigned readings.

9/8: The expectation operator

random variable · PMFs/PDFs/CDFs · expected value of a random variable · integration by parts · linearity of expectation · expectations of functions of random variables · variance · properties of the variance operator

No assigned readings.

9/10: Multivariate calculus: partial and total derivatives

multivariate functions · partial derivatives · linear approximations · total differences · total differentials · total derivatives · (differenatiating along) curves · directional derivatives

SimBlu 13.1; 14.1-14.6.

9/15: Multivariate calculus: higher order derivatives and integration

gradients · the Hessian matrix · Young's theorem · multidimensional integrals · Fubini's theorem · integrals with separable functions · the Gaussian integral

SimBlu 14.8.

9/17: Joint distributions and further issues with the expectation operator

joint distributions of random variables · marginal distribution · conditional distribution · conditional expectation · independent random variables · covariance · correlation · variance of sums of random variables · variance-covariance matrix · multivariate normal distribution

No assigned readings.

9/22: Taylor series expansion and other approximations

local linear approximation · Taylor series · kth order Taylor polynomials · Maclaurin series · the Delta method

SimBlu 2.7; 30.2-30.3.

II: Linear Algebra

9/24: Three images of linear algebra

(systems of) linear equations · spanning vectors · linear (in)dependence · transformation matrices · storing data · projections

9/29: Systems of Linear Equations I

substitution · Gauss-Jordan elimination · augmented matrix · matrix row operations · (reduced) row echelon form · basic and free variables · rank · nonsingular matrices · linear independence · results on existence and number of solutions

SimBlu 7.1-7.4.

10/1: Systems of Linear Equations II

matrix inversion · noninvertible · determinants · minors/cofactors/adjoints · properties of inverses and determinants

SimBlu 8.1-8.4; 9.1-9.2.

10/6: The Algebra of Vectors

vector operations · length and direction of vectors · inner product · inner product and angle · orthogonality · triangle inequality · parametric representation of a line · linear combinations

SimBlu 10.1-10.6.

10/8: Vector spaces

Linear independence · spanning vectors · basis vectors · dimension of a space · vector spaces and subspaces · row space · column space · null space · the fundamental theorem of linear algebra

SimBlu 11.1-11.3; 27.1-27.5.

10/15: Eigendecomposition

eigendecomposition of vcov · eigenvalues · eigenvectors · characteristic equations/roots · systems of linear difference equations · diagonalization

SimBlu 23.1-23.3; 23.7.

10/20: Quadratic forms

monomials · quadratic forms · positive (negative) definiteness · positive (negative) semidefiniteness · indefiniteness · (leading) principal minors · tests for definiteness using leading principal minors · tests for definiteness using eigenvalues

SimBlu 13.3; 16.1-16.2.

10/22: In-class midterm examination

10/27: Implicit Functions

explicit functions · implicit functions · implicit function theorem

SimBlu 15.1-15.2.

10/29: Ordinary differential equations

ordinary differential equations · autonomous DEs · nth order DEs · the logistic curve · solving first order DEs · separable equations · direction field · stationary or equilibrium points

SimBlu 24.1-24.4.

III: Optimization

11/3: Sequences and Series

sequences · limits · convergence/divergence · accumulation point · limsup/liminf · Cauchy sequence · monotonicity · series · power series

Sun 1.2.1-1.2.6.

11/5: Detour: Sequences of Random Variables I

sequences of random variables · convergence in probability · convergence in mean squared · consistent estimators

No assigned reading.

11/10: Detour Cont.: Sequences of Random Variables II

convergence in distribution · moment generating function (MGF) · (lindeberg-levy) central limit theorem

No assigned reading.

11/12: Compactness and Optimization

sets · set relations (equality, subset, disjoint, union, intersections, complements) · set operations (products, differences) · correspondence · function · bijection · metric spaces · open/closed sets · boundedness · compactness · continuity · upper/lower contour sets · the weierstrass theorem · open coverings

Sun 1.2.7-1.2.10; 3.1-3.2.

11/17: Unconstrained (and Multivariate) Optimization

unconstrained optimization · local/global maxima/minima · unconstrained local maxima · first order conditions (FOCs) · critical points · second order conditions · gradient vector · hessian matrix

Sun 4.1-4.5.

11/19: Convexity and Quasiconvexity

convex sets · epigraph/subgraph · convex/concave functions · tests for concavity · implications of concavity (for optimization)

Sun 7.1-7.5.

11/24: Contrained Optimization I: The Lagrangian

the Lagrangian · Lagrange multipliers · constraint requirement · checking for maxima/minima

Sun 5.1-5.5

12/1: Constrained Optimization II: Inequality Constraints
Sun 6.1-6.3.

12/3: Fixed point theorems
Sun 9.1-9.2; 9.4.

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